

The generating functions under view are *real* in the sense of Cayley and Sylvester. Enumerating generating functions of various kinds are obtained by assigning equalities between the suffixed capitals

$$X_1, X_2, \dots, X_s.$$

Putting, *e.g.*,  $X_1 = X_2 = \dots = X_s = x$ ,

we obtain the function which enumerates by the coefficient of  $x^n$ , in the ascending expansion, the numbers of solutions for which

$$\alpha_1 + \alpha_2 + \dots + \alpha_s = n.$$

It will be gathered that the note of the following investigation is the importation of the idea that the solution of any system of equations of the form

$$A_1\alpha_1 + A_2\alpha_2 + A_3\alpha_3 + \dots + A_s\alpha_s \geq 0$$

(all the quantities involved being integers) is a problem of partition analysis, and that the theory proceeds *pari passu* with that of the linear Diophantine analysis.

“On the Boiling Point of Liquid Hydrogen under Reduced Pressure.” By JAMES DEWAR, M.A., LL.D., F.R.S. Received and Read December 15, 1898.

The June number of the ‘Proceedings of the Chemical Society’ contains a paper by the author on “The Boiling Point and Density of Liquid Hydrogen.” A resistance thermometer made of fine platinum wire, called No. 7 Thermometer, was used in the investigation. It had been carefully calibrated, and gave the following resistances at different temperatures :—

Temperature.	Resistance. Ohms.
+ 99·1° C.	7·337
+ 75·3	6·859
+ 51·4	6·388
+ 25·7	5·857
+ 0·7	5·338
– 78·2	3·687
– 182·6	1·398
– 193·9	1·136
– 214·0	0·690

The zero of the thermometer in platinum degrees was  $-263\cdot27^\circ$ . Mr. J. D. Hamilton Dickson, M.A., Fellow of Peterhouse, who contributed a paper to the ‘Phil. Mag.’ for June, 1898, on “The Reduction

to Normal Air Temperature of the Platinum Thermometers," used in the low temperature researches of Professor Fleming and the author, has been good enough to calculate a special formula for this thermometer No. 7. He finds the formula

$$(R + 43.958933)^2 = 2.03596488 (t + 1193.1460)$$

expresses the relation between the resistance and temperature in centigrade degrees. This expression gives a probable error of only  $0.16^\circ \text{C.}$  over a range of more than  $300^\circ \text{C.}$  When this thermometer was placed in boiling hydrogen, the resistance became  $0.129 \text{ ohm,}$  and remained constant at this value. Calculated into the Dickson formula, this value of the resistance corresponds to a temperature of  $-238.4^\circ \text{C.}$  If we assume the resistance reduced to zero, then the temperature registered by the thermometer ought to be  $-244^\circ \text{C.}$  At the boiling point of hydrogen, therefore, if the law correlating resistance and temperature can be pressed to its limits, a lowering of the boiling point of hydrogen by  $5^\circ$  or  $6^\circ \text{C.}$  would produce a condition of affairs where the platinum would have no resistance, or become a perfect conductor. Now we have every reason to believe that hydrogen, like other liquids, will boil at a lower temperature the lower the pressure under which it is volatilised. The question arises, how much lowering of temperature can we practically anticipate. For this purpose we have the boiling point and critical data available from which we can calculate an approximate vapour pressure formula, accepting  $35^\circ \text{ abs.}$  as the boiling point;  $52^\circ \text{ abs.}$  as the critical temperature, and  $19.4 \text{ at.}$  as the critical pressure; then as a first approximation

$$\log p = 6.8218 - \frac{137.9}{T} \text{ mm.} \dots\dots\dots (1).$$

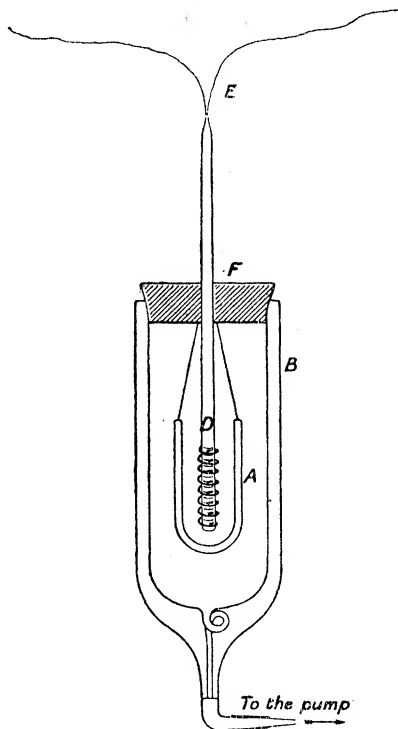
If instead of using the critical pressure in the calculation we assume the molecular latent heat of hydrogen is proportional to the absolute boiling point, then from a comparison with an expression of the same kind, which gives accurate results for oxygen tensions below one atmosphere, we can derive another expression for hydrogen vapour pressures, which ought to be applicable to boiling points under reduced pressure.

The resulting formula is

$$\log p = 7.2428 - \frac{152.7}{T} \text{ mm.} \dots\dots\dots (2).$$

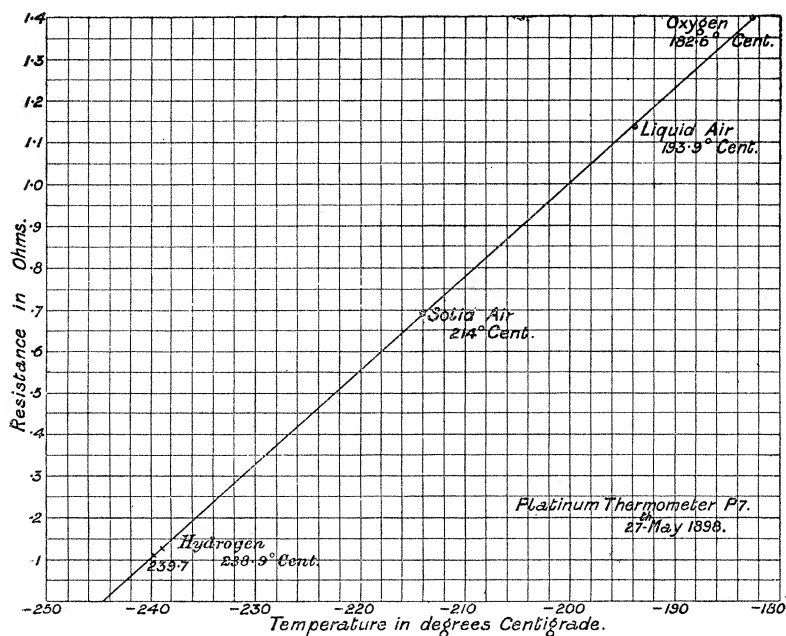
Now formula (1) gives a boiling point of  $25.4^\circ \text{ abs.}$  under a pressure of  $25 \text{ mm.,}$  whereas the second equation (2) gives for the same pressure  $26.1^\circ \text{ abs.}$  As the absolute boiling point under atmospheric pressure is  $35^\circ$ , both expressions lead to the conclusion that ebullition

under 25 mm. pressure ought to reduce the boiling point some  $10^{\circ}$  C. For some time experiments have been in progress with the object of determining the temperature of hydrogen boiling under about 25 mm. pressure, but the difficulties encountered have been so great, and repeated failures so exasperating, that a record of the results so far reached becomes advisable. The troubles arise from the conduction of heat by the leads; the small latent heat of hydrogen volume for volume as compared with liquid air; the inefficiency of heat isolation and the strain on the thermometer brought about by solid air freezing on it and distorting the coil of wire. In many experiments the result has been that all the liquid hydrogen has evaporated before the pressure was reduced to 25 mm., or the thermometer was left imperfectly covered. The apparatus employed will be understood from the figure.



The liquid hydrogen collected in the vacuum vessel A was suspended in a larger vessel of the same kind B, which is so constructed that a spiral tube joins the inner and outer test-tubes of which B is made, thereby making an opening into the interior at C. The resistance thermometer D and leads E pass through a rubber cork F, and the

exhaustion takes place through C. In this way the cold vapours are drawn over the outside of the hydrogen vacuum vessel, and this helps to isolate the liquid from the connective currents of gas. To effect proper isolation the whole apparatus ought to have been immersed in liquid air under exhaustion. Arrangements of this kind add to the complication, so in the first instance the liquid was used as described. The liquid hydrogen evaporated quietly and steadily under a pressure of about 25 mm. of mercury, without the least appearance of solidification or loss of mobility; still remaining clear and colourless to the eye. Naturally the liquid does not last long, so the resistance has to be taken quickly. Just before the reduction of pressure began, the resistance of the thermometer was 0.131 ohm. This result compares favourably with the former observation on the boiling point, which gave a resistance of 0.129 ohm. On reducing the pressure, the resistance diminished to 0.114 ohm, and kept steady for some time. The lowest reading of resistance was 0.112 ohm. This value corresponds to  $-239.1^{\circ}\text{C.}$ , or only one degree lower than the boiling point at atmospheric pressure, whereas the temperature ought to have been reduced some  $10^{\circ}\text{C.}$  or in any case  $5^{\circ}$  under the assumed exhaustion. The position of the observation on the curve of the relation of temperature and resistance for No. 7 thermometer is shown on the accompanying diagram. The question arises then as to what is the explanation of



this result? Has the platinum resistance thermometer arrived at a limiting resistance about  $35^{\circ}$  abs., so that at a lower temperature it refuses to change in resistance, the curve having become practically asymptotic to the axis of temperature? On the other hand, has the influx of heat by the leads, and the correction on account of this change of resistance, become so great as to vitiate the results at these excessively low temperatures? Again, it may be suggested that the thermometer was not properly cooled, or that the liquid hydrogen does not lower in temperature to any marked extent under exhaustion like other liquids. All these conjectures can only be set at rest by a repetition of the experiments with a new thermometer of much higher initial resistance, and under conditions of better heat isolation. No blunder having been detected in the observations, for the present we must assume that the platinum resistance thermometer No. 7 acts in the manner described. It would be premature to discuss the inferences to be drawn from these results until they are confirmed on another variety of platinum wire made into a resistance thermometer. But as this will involve the use of considerable quantities of liquid hydrogen, it will take some time to complete the investigation.

The same kind of anomaly appears in the case of the use of a thermojunction at these low temperatures, but this is a separate matter, and must be dealt with in a further communication.

I am indebted to Mr. J. E. Petavel for assistance in the electrical measurements, and also to Mr. Robert Lennox and Mr. Heath for their general help in the conduct of the experiments.

“Application of Liquid Hydrogen to the Production of High Vacua, together with their Spectroscopic Examination.” By JAMES DEWAR, M.A., LL.D., F.R.S. Received and Read December 15, 1898.

As an illustration of the extraordinary power of the new cooling agent—liquid hydrogen, the extreme rapidity with which high vacua can be produced by its use is, perhaps, one of the most striking. The absolute boiling points of hydrogen, oxygen, and chlorine are respectively  $35^{\circ}$ ,  $90^{\circ}$  and  $240^{\circ}$ , in other words oxygen boils at a temperature two and a half times higher than liquid hydrogen, and liquid chlorine similarly at two and a half times that of liquid oxygen. From this we infer that liquid hydrogen as a cooling agent ought to be relative to liquid air as effective as the latter is compared to that of liquid chlorine. Now chlorine at the temperature of boiling oxygen is a hard solid, some  $80^{\circ}$  below its melting point, and in this condition has an excessively feeble vapour pressure. When liquid hydrogen freezes air out of a sealed tube by immersing the end in the liquid, it is to be